

TEICHMÜLLER DYNAMICS, MAPPING CLASS GROUPS AND APPLICATIONS
GRENOBLE JUNE 25 - 29, 2018

Paul Apisa (Chicago) *Marked points in genus two and beyond*

In the principal stratum in genus two, McMullen observed that something odd happens - there is only one nonarithmetic Teichmüller curve - the one generated by the decagon. This strange phenomenon begets another - a primitive translation surface in genus two admits a periodic point that is not a Weierstrass point or zero only if it belongs to the golden eigenform locus. In this talk, we will explain how to leverage results of Mirzakhani-Wright to study the orbit closures of translation surfaces with marked points and sketch a proof of the previously mentioned result in genus two. We will also explain how the result in genus two proves another uniqueness results - that there is at most one nonarithmetic rank two orbit closure in the minimal stratum in genus four - the one discovered by Eskin-McMullen-Mukamel-Wright.

Dawei Chen (Boston College) *Are affine invariant submanifolds affine?*

We study the question whether affine invariant submanifolds arising from Teichmüller dynamics are affine varieties in the sense of algebraic geometry.

Diana Davis (Swarthmore College) *Periodic paths on the pentagon*

Mathematicians have long understood periodic trajectories on the square billiard table. In the present work, we describe periodic trajectories on the regular pentagon – their geometry, symbolic dynamics, and group structure. Some of the periodic trajectories exhibit a surprising "dense but not equidistributed" behavior. I will show pictures of periodic trajectories, which are very beautiful. This is joint work with Samuel Lelièvre and Barak Weiss.

Benjamin Dozier (Stanford) *Translation surfaces with multiple short saddle connections*

I will discuss a proof in progress of a natural conjecture on the volume of a subset of an affine invariant manifold (in a stratum of translation surfaces) composed of surfaces with multiple independent short saddle connections. This is a strengthening of the regularity result proved by Avila-Matheus-Yoccoz. A key tool is the new compactification of strata due to Bainbridge-Chen-Gendron-Grushevsky-Møller, which gives a good picture of how a translation surface can degenerate. The techniques are expected to be useful for other problems concerning affine invariant manifolds.

Alex Eskin (Chicago) *Counting closed geodesics on translation surfaces*

We show that the number of closed geodesics in the flat metric on a translation surface of length at most R is asymptotic to $e^{hR}/(hR)$. This is joint work with Kasra Rafi.

Charles Fougere (Skoltech) *Diffusion rate for windtree models*

Recent results on windtree models with polygonal obstacles have linked their diffusion rate with Lyapunov exponents in strata of quadratic differentials. The proofs of these theorems follow from the numerous symmetries of the studied examples. In this talk I will present a general method for this purpose in a more general setting using the latest advances on the structure of the orbit closure of translation surfaces and their Hodge bundle.

These techniques give a first step for understanding the asymptotic behaviour of the diffusion rate of regular polygons and other shapes. I will also present some computer experiments and conjectures on these.

Vaibhav Gadre (Glasgow) *Effective convergence of ergodic averages and cusp excursions of geodesics on moduli spaces*

We survey some applications of effective convergence of ergodic averages to the analysis of cusp excursions of typical geodesics on moduli spaces. This will cover Teichmüller geodesics, Weil-Petersson geodesics and geodesics typical for harmonic measures arising from random walks on mapping class groups.

Rodolfo Gutierrez (Paris 13) *Quaternionic monodromies of the Kontsevich–Zorich cocycle*

The monodromy group of a translation surface M is the Lie group spanned by all symplectic matrices arising from the homological action of closed loops at M (inside its embedding orbit closure). In the presence of zero Lyapunov exponents, Filip showed that these groups are —up to compact factors and finite-index subgroups— constrained at the level of Lie groups representations: they are either special quaternionic orthogonal groups $SO^*(2n)$ for odd n , special unitary groups $SU(p, q)$ for $p > q$, or exterior powers of $SU(p, 1)$. Nevertheless, it does not follow from these constraints that every group in this list is realizable as the monodromy group of some translation surface.

By the work of Avila–Matheus–Yoccoz, it is known that every $SU(p, q)$ is realizable. Moreover, the work of Filip–Forni–Matheus shows an example that realizes $SO^*(6)$, which coincides with the second exterior power of $SU(3, 1)$.

In this talk, we show that the groups $SO^*(2n)$ are realizable as monodromy groups for infinitely many odd values of n . To this end, we find concrete examples of square-tiled surfaces constructed as quaternionic covers —generalizing the construction by Filip–Forni–Matheus— having such groups as their monodromies (up to compact factors and finite-index subgroups). Our construction is valid for every n in the congruence class of $3 \pmod{8}$, up to a zero-density set.

Christopher Judge (Indiana University) *Systoles in translation surfaces*

I will discuss joint work with Hugo Parlier concerning the shortest noncontractible loops—'systoles'—in a translation surface. In particular, we provide estimates (some sharp) on the number of systoles (up to homotopy) in the strata $H(2g-2)$ and the stratum $H(1,1)$. We also determine the maximum systolic ratio (length squared/area) in $H(2g-2)$, and we give a conjectural value for maximum systolic ratio in $H(1,1)$.

Anna Lenzhen (Rennes) *Limit sets of Teichmüller geodesic rays in the Thurston boundary of Teichmüller space*

H. Masur showed in the early 80s that almost every Teichmüller ray converges to a unique point in PMF. It is also known since a while that there are rays that have more than one accumulation point in the boundary. I will give an overview of what is understood so far about the limit sets of Teichmüller rays, mentioning some recent progress. For example, I will mention recent joint work with K. Rafi and B. Modami where we give a construction of a ray whose limit set in PMF is a d -dimensional simplex.

Livio Liechti (Paris 7) *Minimal dilatations on nonorientable surfaces*

We discuss the problem of finding the minimal dilatation among pseudo-Anosov mapping classes on a fixed closed surface. In particular, for every nonorientable closed surface of even genus, we consider a simple candidate which conjecturally minimises the dilatation among pseudo-Anosov mapping classes with an orientable invariant foliation. This is joint work with Balázs Strenner.

Stefano Marmi (Pisa) *Diophantine interval exchange maps*

Curtis T McMullen (Harvard) *Billiards, quadrilaterals and moduli spaces*

Olga Paris-Romaskevich (Rennes) *Triangle tiling billiards : a hopscotch guided by an interval exchange transformation with flips*

Tiling billiards is a dynamical system in which a billiard ball moves through the tiles of some fixed tiling in a way that its trajectory is a broken line, with breaks admitted only at the boundaries of the tiles. One can think about this system as a movement of the refracted light.

In this talk, I will speak about a specific kind of such billiards: negative triangle tiling billiards introduced in the work by P. Baird-Smith, D. Davis, E. Fromm and S.Iyer. The tiling is a periodic tiling of a plane by congruent triangles (obtained from the standard equilateral triangle tiling by linear transformation). The law of reflection in any side is Snell's law with the coefficient of refraction equal to -1.

This system is, in a simple and magical way, related to interval exchange transformations with flips. I will explain this relationship and some results that follow from it.

First, almost all the trajectories of such tilings are either closed or escape linearly to infinity. Second, there exists an interesting measure zero set of parameters for which the trajectories escape to infinity in a non-linear fashion, and approach fractals. This set is parametrised by the Rauzy gasket.

My talk will be based on a joint work with Pascal Hubert, some computer simulations by Paul Mercat, and the article of D. Davis and her coauthors.

Irene Pasquinelli (Durham) *Deligne-Mostow lattices and cone metrics on the sphere*

Finding lattices in $PU(n,1)$ has been one of the major challenges of the last decades. One way of constructing a lattice is to give a fundamental domain for its action on the complex hyperbolic space.

One approach, successful for some lattices, consists of seeing the complex hyperbolic space as the configuration space of cone metrics on the sphere and of studying the action of some maps exchanging the cone points with same cone angle.

In this talk we will see how this construction can be used to build fundamental polyhedra for all Deligne-Mostow lattices in $PU(2,1)$.

Anja Randecker (Toronto) *Average diameter of translation surfaces*

For a given stratum, we can consider the average of the diameters over all translation surfaces in the stratum. In joint work with Howard Masur and Kasra Rafi, we show that for the minimal strata, this number goes to zero when the genus goes to infinity.

Ramanujan Santharoubane (Virginia) *Quantum representations of surface groups*

I will show how we can produce exotic representations of surface groups from the Witten-Reshetikhin-Turaev TQFT. These representations have infinite images and give points on character varieties that are fixed by the action of the mapping. Moreover we can approximate these representations by representations into finite groups in order to build exotic regular finite covers of surfaces. These covers have the following property: the integral homology is not generated by pullbacks of simple closed curves on the base. This is joint work with Thomas Koberda.

Saul Schleimer (Warwick) *An introduction to veering triangulations*

Singular euclidean structures on surfaces are a key tool in the study of the mapping class group, of Teichmüller space, and of kleinian three-manifolds. François Guéritaud, while studying work of Ian Agol, gave a powerful technique for turning a singular euclidean structure (on a surface) into a triangulation (of a three-manifold). We will give an exposition of some of this work from the point of view of Delaunay triangulations for the L^∞ -metric. We will review the definitions in a relaxed fashion, discuss the technique, and then present applications to the study of strata in the space of singular euclidean structures. If time permits, we will also discuss the naturally occurring algorithmic questions.

Alex Wright (Stanford) *Nearly Fuchsian surface subgroups of finite covolume Kleinian groups*

We will present joint work with Jeremy Kahn proving that any complete cusped hyperbolic three manifold contains many "nearly isometrically immersed" closed hyperbolic surfaces.

Öykü Yurttaş (Dicle) *Algorithms for multicurves with Dynnikov coordinates*

Multicurves have played a fundamental role in the study of mapping class groups of surfaces since the work of Dehn. A beautiful method of describing such systems on the n -punctured disk is given by the Dynnikov coordinate system. In this talk we describe polynomial time algorithms for calculating the number of connected components of a multi curve, and the geometric intersection number of two multicurves on the n -punctured disk, taking as input their Dynnikov coordinates. This is joint work with Toby Hall.
