Some Problems.

① Show that for rational billiards each trajectory travels in a finite number of directions.

② If we unfold a nbd of a singular point with cone angle $\frac{p\pi}{q}$ show that we get a p to 1 map in a neighborhood of the singular point.

③ Show that the probability that a random pair of integers is relatively prime is $\frac{1}{\zeta(2)}$ where $\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2}$.

(4) Show that the geodesic flow on a translation surface extends continuously to a singular point if and only if the cone angle is equal to 2π.

(5) Show that the billiard flow has a continuous extension a vertex of a polygon if the cone angle has the form $\frac{\pi}{q}$.

⑥ Show that a rational polygon tiles the plane by reflection if and only if it corresponds to a translation surface of genus 1.

⑦ Find the rational triangles that tile the plane by reflection.

8. Show that gluing together opposite sides of the regular octagon gives a surface of genus 2 with one cone point with cone angle 6π

9. Show that gluing together opposite sides of a decagon gives a surface of genus 2 with 2 cone points with cone angles 4π each.

(10) Let us assert that the curvature of a translation surface at a singular point with cone angle c is $k = 2\pi - c$. Show that with this defintion of curvature the Gauss-Bonnet theorem holds for translation surfaces.

(11) If a translation surface is given by a holomorphic 1-form then the order of the zero is $\frac{-K}{2\pi}$ $\left(= \frac{c_j - 2\pi}{2\pi} \right)$.

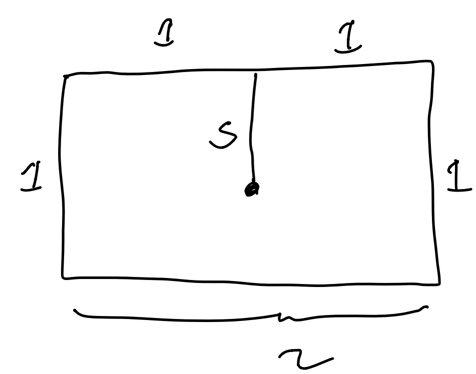
Given a lattice $\Lambda \subset \mathbb{R}^2$ and a basis v_1, v_2 so $\Lambda = \{ m v_1 + n v_2 : m, n \in \mathbb{Z} \}$ with $v_1 = \begin{bmatrix} a \\ b \end{bmatrix}$, $v_2 = \begin{bmatrix} c \\ d \end{bmatrix}$.

We associate Λ with the matrix $[v_1 \ v_2] = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$.

(12) Show that changing the basis of a lattice corresponds to the right action of $SL_{\pm}(2,\mathbb{Z})$

(13) Show that changing a lattice on torus by applying the geometric action of a matrix A corresponds to changing the matrix representation by left multiplication by A.

(14) Consider the translation surface associated to the slit rectangle. Show that if the slit length s is rational then this surface is square tiled while if the slit length is irrational then the $SL(2,\mathbb{R})$ orbit closure is E_4.



(15) Show that the collection of "parallelogram tiled surfaces" is closed and $SL(2,\mathbb{R})$ invariant.